Effect of Dissipation and Compression Work on the Eddy Conductivity Calculated from Experimental Temperature Data for Gases

JOSE A. BLANCO and WILLIAM N. GILL
Clarkson College of Technology, Potsdam, New York

The effect of dissipation and compression work on the eddy conductivity calculated from experimental temperature distribution data for gases is examined in detail, and it is shown that these effects are significant even in relatively low Reynolds number flows, say greater than 30,000. The inclusion of dissipation and compression work in the energy equation for gaseous flows is shown to lead to the determination of symmetrical eddy diffusivities from experimental data.

The temperature distribution in the fully developed thermal region for a fluid flowing turbulently between two walls maintained at uniform but different temperatures is a function of the transverse coordinate only, and therefore both the wall temperature and heat flux are independent of the axial coordinate. The heat entering at one wall is effectively "conducted" away by the fluid and then lost to the other wall; the fluid is heated by contact with one wall and cooled by contact with the other. This mode of convective heat transfer is particularly favorable for determining the behavior of the eddy conductivity at the center of the channel, as was recognized by Harrison and Menke (*9*) and Corcoran, Sage, and co-workers (*3* to *6*, *11*, *13*, *14*, *16*).

Dissipation, an energy source, increases the temperature of the fluid and consequently alters the temperature gradient distribution for heating and cooling in opposite directions. A parallel argument holds for the effects of compression work, which behaves as an energy sink. An asymmetrical arrangement is thus particularly interesting because the fluid is simultaneously heated at one wall and cooled at the other, and therefore dissipation and compression work may introduce asymmetries in the temperature gradient distribution even if the fluid properties are assumed independent of temperature. For these reasons, the asymmetrical arrangement discussed above, which was studied experimentally by Corcoran, Sage, and co-workers (*3* to *6*, *11*, *13*, *14*, *16*) with the objective of determining the eddy diffusivity distribution, is also particularly interesting from the point of view of investigating the presence of dissipation and compression work and the extent to which these effects may intervene in the heat transfer process.

Previous theoretical developments (*12*, *17*, *18*) indicate that for gaseous flows, dissipation and compression work may affect measured temperature profiles. The effects of compression work on the temperature distribution are important for compressible flows whenever the dissipative effects cannot be neglected; the latter are of the same order of magnitude as the other terms in the equation of energy when the Eckert number is on the order of unity. Dissipation in turbulent gaseous flows was introduced by Venezian and Sage (*19*) in an attempt to correct the asymmetries reported in the eddy conductivity evaluated from the experimental temperature distributions of Corcoran, Sage et al. Compression work, however, has not received attention, although the combined effects of dissipation and compression work may help account for these asymmetries in the eddy conductivity distribution.

The concept of eddy diffusion loses its usefulness if the total diffusivity is affected by the presence of sources, sinks, or any secondary effects which have the net result of altering its numerical values. This is because such dependencies would preclude extrapolating the results obtained from one set of experiments to another. They would also impair the possibility of obtaining solutions to problems of increasing complexity by addition of simpler solutions whose validity is more easily checked. It is thus the objective of this paper to study the effects that dissipation and compression work have upon the temperature profile in asymmetrically heated systems, and, conversely, to propose a model such that the eddy coefficients calculated from experimental data are not affected by the presence of compression or dissipation work.

ANALYSIS

For the configuration shown in Figure 1 and assuming that the fluid properties are independent of temperature, the equation of motion for fully developed turbulent flow is

$$\frac{dP}{dx} = \frac{d}{dy}(\mu + \rho \varepsilon_M)\frac{du}{dy}, \quad -R_e \leq y \leq R_e \tag{1}$$

The effects of compressibility on the velocity distribution are assumed to be negligible. This is a tenable assumption for low Mach number flows (*15*), even though a fully developed velocity distribution is not compatible with compressibility effects (*7*).

The equation of energy is

$$\frac{d}{dy}(k + \rho C_p\, \varepsilon_H)\frac{dT}{dy} + (\mu + \varepsilon_M)\left(\frac{du}{dy}\right)^2 + u\frac{dP}{dx} = 0 \tag{2}$$

where the term

$$(\mu + \varepsilon_M)\left(\frac{du}{dy}\right)^2$$

Jose A. Blanco is with the International Nickel Co. of Canada Ltd., Port Colborne, Ontario.

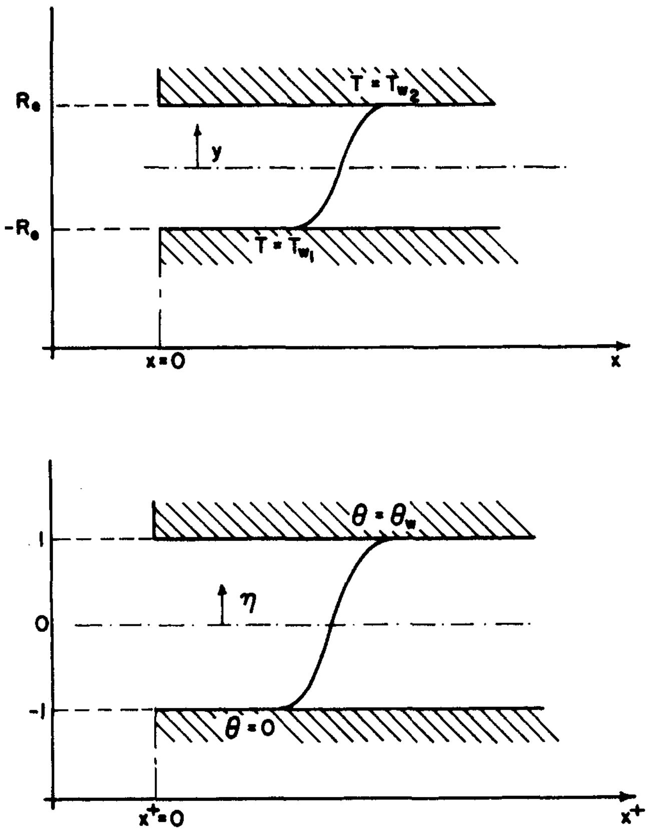

Fig. 1. Schematic diagram of parallel plate duct with unequal temperatures at both walls.

represents molecular and turbulent dissipation as was shown by Venezian and Sage (*19*), and $u(dP/dx)$ is assumed to represent the effects of compressibility. Intuitively, one may expect that the effects of compressibility on the velocity distribution affect the temperature profile to a lesser extent than they affect it by serving as forcing functions in the energy equation. Equations (1) and (2) apply to laminar flow when setting $\varepsilon_H = \varepsilon_M = 0$.

Compression work does not appear in the energy equation for incompressible liquids since then $C_p \simeq C_v$ (1) and therefore this term appears only for gases, which usually have a Prandtl number on the order of unity. Thus a parameter s will be introduced such that $s = 1$ for gases and $s = 0$ for liquids. Then, Equation (2) can be written in dimensionless coordinates with the aid of Equation (1) as follows:

$$\frac{d}{d\eta} f \frac{d\theta}{d\eta} + N_E \frac{dP^+}{dx^+} \left\{ \eta \frac{dw}{d\eta} + sw \right\} = 0, \quad -1 \leq \eta \leq 1 \tag{3}$$

where

$$f = 1 + \frac{N_{Pr}}{N_{Pr_T}} \frac{\varepsilon_M}{\nu}$$

and

$$s = 1 \text{ for gases}$$

$$s = 0 \text{ for liquids}$$

For uniform wall temperatures the boundary conditions are

$$\theta(-1) = 0$$

$$\theta(1) = \theta_w$$

Then, the solution to Equation (3) can be written, for convenience, as

$$\theta(\eta) = \theta_1(\eta) + \theta_2(\eta) \tag{4}$$

where

$$\frac{d}{d\eta} f(\eta) \frac{d\theta_1}{d\eta} = 0 \tag{5}$$

with

$$\theta_1(-1) = 0, \quad \theta_1(1) = \theta_w \tag{6}$$

and

$$\frac{d}{d\eta} f(\eta) \frac{d\theta_2}{d\eta} - N_E \left| \frac{dP^+}{dx^+} \right| \left\{ \eta \frac{dw}{d\eta} + sw \right\} = 0 \tag{7}$$

with

$$\theta_2(-1) = \theta_2(1) = 0 \tag{8}$$

Integrating Equations (5) and (7), one obtains

$$\theta_1(\eta) = \frac{1}{2} \theta_w \left\{ 1 + \frac{\int_0^\eta \frac{1}{f} d\eta}{\int_0^1 \frac{1}{f} d\eta} \right\} = \frac{d\theta_1(1)}{d\eta} \int_{-1}^{\eta} \frac{1}{f} d\eta \tag{9}$$

and

$$\theta_2(\eta) = N_E \left| \frac{dP^+}{dx^+} \right| \int_1^\eta \frac{\eta w + (s-1) \int_0^\eta w \, d\eta}{f} d\eta \tag{10}$$

$$= \frac{d\theta_2}{d\eta}(1) \int_1^\eta \frac{1}{f} d\eta$$

$$+ N_E \left| \frac{dP^+}{dx^+} \right| \int_1^\eta \frac{\eta w + (s-1) \int_1^\eta w \, d\eta}{f} d\eta \tag{11}$$

Also, from Equation (4)

$$f \frac{d\theta}{d\eta} = f \frac{d\theta_1}{d\eta} + f \frac{d\theta_2}{d\eta} = N_E \left| \frac{dP^+}{dx^+} \right| \left\{ \eta w + (s-1) \int_1^\eta w \, d\eta \right\} + \frac{d\theta}{d\eta}(1) \tag{12}$$

Note that if $N_E = 0$, that is, if dissipation and compression work effects are negligible, the heat flux is independent of radial position and Equation (12) reduces to

$$f \frac{d\theta}{d\eta} = \frac{d\theta}{d\eta}(1) \tag{13}$$

which defines f. That is, Equation (13) can be used to compute $f(\eta)$ from experimental data only if $N_E = 0$. If compression work and/or dissipation cannot be neglected, Equation (13) leads to conductivity distributions that are in error. This point was first made by Venezian and Sage (*19*) in an attempt to correct the total conductivity distributions reported by Hsu et al. (*11*), who used an equivalent form of Equation (13) to compute $f(\eta)$ from temperature distribution data for flow of air in the configuration of Figure 1. However, Venezian and Sage (*19*) neglected the effects of compression work.

The Temperature Distribution

If one assumes that the eddy conductivity is symmetrical about the center of the channel, Equations (4) and (9) for $N_E = 0$ predict that

$$\theta(0) = \theta_1(0) = \tfrac{1}{2} \theta_w \tag{14}$$

that is, the temperature at the center of the channel is the arithmetic mean of the temperatures at both walls if dissipation and compression work are neglected. However, from Equations (4), (9), and (10), for $N_E \neq 0$, the following cases are possible at the center of the channel:

1. Only dissipation is included ($s = 0$, $N_E \neq 0$):

$$\theta(0) = \frac{1}{2}\theta_w + N_E \left|\frac{dP^+}{dx^+}\right| \int_1^0 \frac{\int_0^\eta \eta \frac{dw}{d\eta} d\eta}{f} d\eta > \frac{1}{2}\theta_w \quad (15)$$

2. Both dissipation and compression work are included ($s = 1$, $N_E \neq 0$):

$$\theta(0) = \frac{1}{2}\theta_w + N_E \left|\frac{dP^+}{dx^+}\right| \int_1^0 \frac{\eta w}{f} d\eta < \frac{1}{2}\theta_w \quad (16)$$

The experimental evidence gathered by Corcoran, Sage and coworkers shows that, in general, the temperature at the center of the channel is such that

$$T_{\mathrm{EXP}}(0) < \frac{T_{w_1} + T_{w_2}}{2}$$

in agreement with the predictions of Equation (16). It seems a plausible proposition that both compression work and dissipation should be considered simultaneously in gaseous flows if the correct trends in the temperature profiles are to be predicted with an eddy diffusivity function which is symmetrical about the centerline of the conduit. It then follows that for gaseous flows, both dissipation and compression work must be considered simultaneously if one attempts to compute the eddy conductivity from experimental temperature data. Venezian and Sage (*19*), in an effort to account for the asymmetries in the eddy conductivity reported by Hsu et al. (*11*), postulated the presence of dissipative effects, which they called "viscous," and sought to correct the eddy diffusion by calculating it from the expression (see Equation (12))

$$f_v \frac{d\theta_{\mathrm{EXP}}}{d\eta}(\eta) = \frac{d\theta_{\mathrm{EXP}}}{d\eta}(1) + N_{E_m} \left|\frac{dP^+}{dx^+}\right| \left[\eta w - \int_1^\eta w\, d\eta\right] \quad (17)$$

where $\frac{d\theta_{\mathrm{EXP}}}{d\eta}(\eta)$ is computed from the experimental temperature distribution, and $\frac{d\theta_{\mathrm{EXP}}}{d\eta}(1)$ is the measured flux at the upper wall of the channel. Then, f_v cannot be an even function of η, since if f_v is even, the temperature profile constructed from Equation (3) satisfies condition (15), that is, the temperature at the center of the channel is higher than the average of the wall temperatures, and this contradicts the experimental data it is supposed to represent. This means that expression (17) cannot eliminate the asymmetries in the eddy distribution calculated from the experimental data. Incidentally, expressions (3) and (15) in the work of Venezian and Sage (*19*) should be integrated between the limits (y_o, y) rather than (y_o, $y_o - y$), but the conclusions in their paper indicate that these discrepancies are merely printing errors.

The Conductivity Distribution

From the foregoing discussion it follows that it is necessary to be very careful when attempting to determine eddy conductivity distributions from experimental temperature profiles. In this analysis, heat sinks, rather than heat sources, seem to account for the experimental deviation of the temperature at the center of the channel while preserving the symmetry of the eddy diffusivity function. Conversely, one must account for the effects of compression work together with dissipation when interpreting experimental temperature distributions for gases. The contributions of dissipation and compression work to the heat flux distribution can be derived from Equation (10). Thus one has

1. Dissipation:

$$\frac{q_{\mathrm{DISS}}}{k\Delta T/R_e} = N_E \left|\frac{dP^+}{dx^+}\right| \left[\eta w - \int_0^\eta w\, d\eta\right] \quad (18)$$

2. Compression work:

$$\frac{q_{\mathrm{COMP}}}{k\Delta T/R_e} = N_E \left|\frac{dP^+}{dx^+}\right| \int_0^\eta w\, d\eta \quad (19)$$

and

$$\frac{1}{k\Delta T/R_e}[q_{\mathrm{DISS}} + q_{\mathrm{COMP}}] = f\frac{d\theta_2}{d\eta} \quad (20)$$

Thus the total conductivity f defined by Equation (20) or (13) can be understood as the "frozen" total conductivity of the fluid, and this concept applies equally well to turbulent or laminar flows. The qualifier "frozen" is used in the sense that f is determined by the nature of the fluid and the type of flow, that is, the parameters N_{Pr}, N_{Pr_T}, and N_{Re}, and is not affected by any other parameter. This idea can also be expressed in a more complete form by writing the heat flux q in the form

$$q = q^c + \sum_i q_i, \quad q^c = f\frac{dT^c}{d\eta}, \quad q_i = f\frac{dT_i}{d\eta}$$

where q^c refers to the flux, and T^c to the temperature, that would result if only convection and conduction were present, and q_i are the fluxes arising from the presence of sources, sinks, species diffusion in multicomponent mixtures, radiant energy, etc. In the context of this work, q^c would be defined by

$$\frac{q^c}{k\Delta T/R_e} = f\frac{d\theta_1}{d\eta} \quad (21)$$

while the q_i are given by Equations (18) and (19). From Equations (20) and (21) one gets

$$f\frac{d\theta}{d\eta} = f\frac{d\theta_1}{d\eta} + f\frac{d\theta_2}{d\eta} = \frac{q_{\mathrm{EXP}}}{k\Delta T/R_e} \quad (22)$$

and from Equation (12)

$$\frac{q_{\mathrm{EXP}}}{k\Delta T/R_e} = f\frac{d\theta_{\mathrm{EXP}}}{d\eta} = \frac{d\theta_{\mathrm{EXP}}}{d\eta}(1) + N_E\left|\frac{dP^+}{dx^+}\right| \left[\eta w + (s-1)\int_1^\eta w\, d\eta\right] \quad (23)$$

Thus if this analysis is taken as the basis for the interpretation of the experimental results discussed earlier, the conductivity is to be computed from Equation (23) with $s = 1$:

$$f\frac{d\theta_{\mathrm{EXP}}}{d\eta}(\eta) = \frac{d\theta_{\mathrm{EXP}}}{d\eta}(1) + N_E\left|\frac{dP^+}{dx^+}\right| [\eta w] \quad (24)$$

and $\theta_{\mathrm{EXP}}(\eta)$ is the experimental temperature profile. In this manner, the energy balance (3) is satisfied and the conductivity is not affected by the presence of dissipation and compression work.

RESULTS

Denoting by f_o the conductivity calculated from Equation (24) with $N_E = 0$ which applies when the contributions of dissipation and compression work are neglected, and is the expression used by Hsu et al. (*11*) to compute total conductivities from the experimental data of Corcoran, Sage et al., one has in general

$$\frac{f(\eta)}{f_o(\eta)} = 1 + \frac{N_E \left| \frac{dP^+}{dx^+} \right| \left[\eta w + (s-1) \int_1^\eta w\, d\eta \right]}{\frac{d\theta_{\text{EXP}}}{d\eta}(1)} \tag{25}$$

where $s = 1$ for gas flows. Thus, once the terms ηw and $\int_1^\eta w\, d\eta$ are known, it is possible to estimate the errors created by neglecting dissipation alone or together with compression work. It is also possible to correct the conductivities reported by Hsu et al. to compensate for dissipation and compression work.

The functions

$$\eta w - \int_1^\eta w\, d\eta$$

$$\eta w \tag{26a, b, c}$$

$$-\int_1^\eta w\, d\eta$$

which correspond to the contributions due to dissipation alone, to dissipation and compression work, and to the difference between these two, respectively, are shown in Figure 2 as a function of position for a Reynolds number of 37,500. These curves were calculated with the eddy viscosity of Gill and Scher (8) and a N_{Pr_T} of unity, which is a reasonable value for gas flows. Note that curve *a* represents the conductivity correction term proposed by Venezian et al. (19); curve *b* represents the correction term proposed in this work; and curve *c* is the difference between these two.

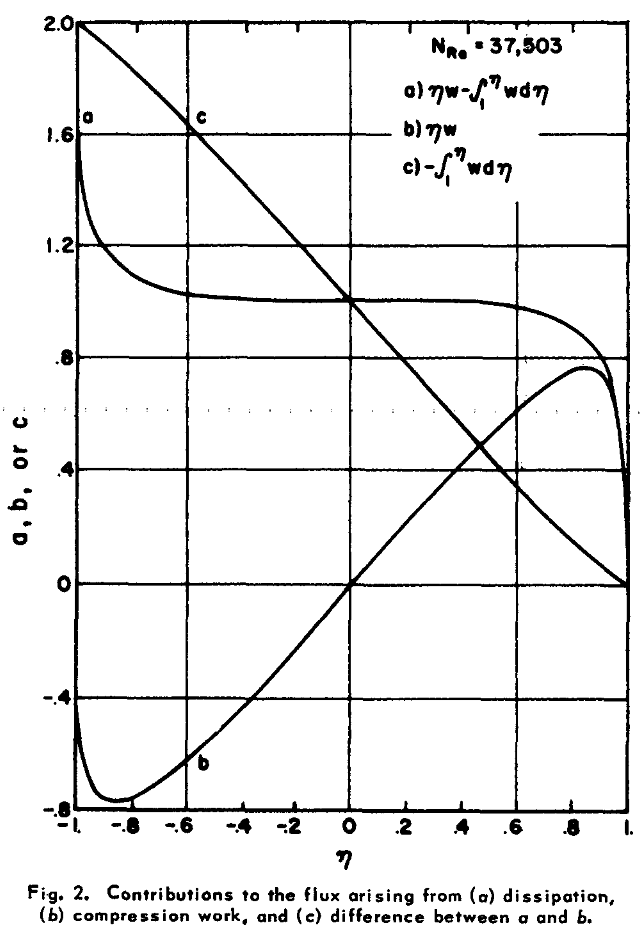

Fig. 2. Contributions to the flux arising from (a) dissipation, (b) compression work, and (c) difference between a and b.

Calculations performed for a Reynolds number of about 57,000 resulted in essentially the same curves as shown in Figure 2 for $N_{Re} = 37{,}500$. In fact, for turbulent flows whose velocity distribution can be approximated by the power law

$$w^+ = \left(\frac{y^+}{y_m^+}\right)^n \quad \frac{1}{10} \le n \le \frac{1}{7}$$

the functions *a*, *b*, *c* of Figure 2 can be expected to remain essentially unchanged. This permits one to estimate the effect that neglecting dissipation and compression work has on the heat flux and conductivity distributions in the region of $x^+ \longrightarrow \infty$ for a wide range of parameters. For this purpose, one can write

$$N_E \left| \frac{dP^+}{dx^+} \right| = N_E\, N_{Pr} \frac{y_m^+}{u_B^+}$$

As an example, consider flow of air ($N_{Pr} = 0.72$) at $N_{Re} = 50{,}000$ with $\theta_w = 2$ and $N_E = .1$, which corresponds to the range of parameters experimentally investigated by Corcoran, Sage et al. The maximum value in curve *b*, Figure 2, is 0.8, and the maximum error in the reduced temperature gradient $d\theta/d\eta$ arising from neglecting dissipation and compression work is

$$\frac{0.8 N_E \left| \frac{dP^+}{dx^+} \right|}{0.8 N_E \left| \frac{dP^+}{dx^+} \right| + \theta_w \Lambda_1} \sim \frac{1.75}{1.75 + 25.2} \sim 6.5\%$$

where the values of y_m^+, u_B^+, and Λ_1 were taken from reference 2.

Doubling the Reynolds number to 100,000 increases the Eckert number by a factor of 4. Then, the maximum error in $d\theta/d\eta$ arising from neglecting dissipation and compression work would be

$$\frac{12.5}{12.5 + 43.8} \sim 22\%$$

This is a rather large error and indicates the need of including dissipation and compression work for this particular example. This result is not very surprising since a Reynolds number of 100,000 for air in the experimental apparatus of Corcoran, Sage et al., which served as model for this analytical work, corresponds to a rather high velocity of about 200 ft./sec. The two examples given above illustrated that significant errors may be involved when neglecting the contributions of dissipation and compression work.

The general approach suggested in this paper implies a mechanistic interpretation of the concept of eddy conductivity. When applied with due care, this permits one to predict the trend of the experimental irregularities reported in the experimental work of Corcoran, Sage, and co-workers and at the same time the simplicity that results from an even eddy diffusivity is maintained. This can be seen, for example, in Figures 3 to 7, where the temperature profiles calculated from Equation (3) with the eddy diffusivity of Deissler as modified by Hatton and Quarmby (10) are plotted versus reduced radial distance for values of the parameters that closely approximate the experimental values of Cavers et al. (3).

The influence of dissipation and compression work on the temperature profiles is seen in Figures 3 to 7 to increase with Reynolds number (that is, Eckert number, since all dimensions in the system remain constant except the velocity). At $N_{Re} \sim 36{,}000$, the contribution of dissipation and compression work to the bulk temperature is about 2%; at $N_{Re} \sim 56{,}000$, it increases to about 5%. However, one should be cautious in extrapolating to higher Reynolds numbers because, as the N_{Re} increases, the compressibility effects become more important and may invalidate the assumption that the flow is incompressible in calculating

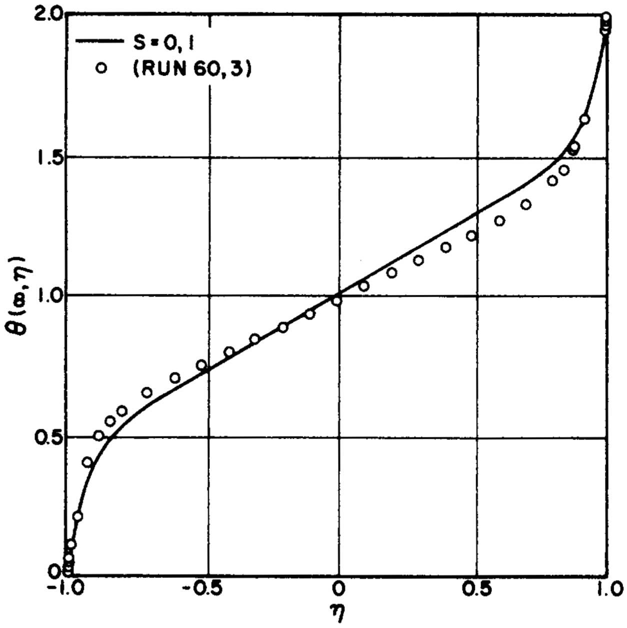

Fig. 3. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures with dissipation ($N_E \neq 0$, $S = 0$) and dissipation and compression work ($N_E \neq 0$, $S = 1$), and the experimental data of reference *3* for air at $N_{Re} = 8{,}122$ Eddy viscosity of Deissler as modified by Hatton et al. (*10*), with $N_{Pr_T} = 1$.

$N_{Re} = 8{,}135$ $\quad N_E = 0.0012$ $\quad N_{Pr} = 0.72$

	$S = 0$	$S = 1$
N_{Nu_1}	20.54	20.53
N_{Nu_2}	20.49	20.51
θ_B	1.	1.

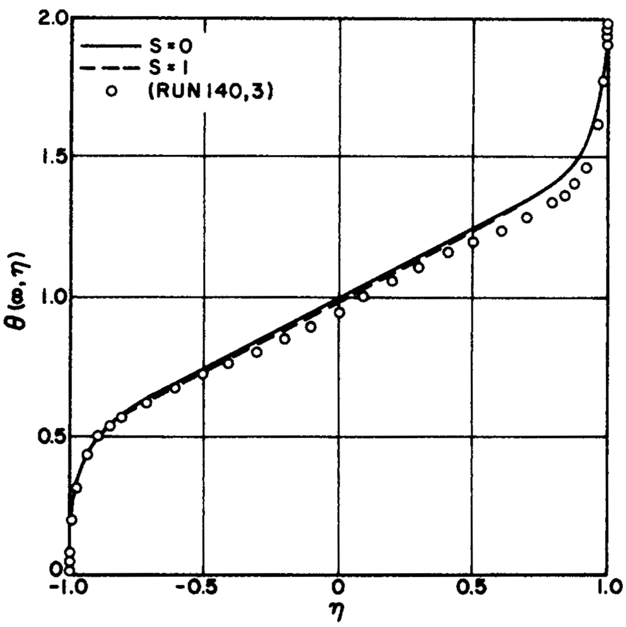

Fig. 4. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures with dissipation ($N_E \neq 0$, $S = 0$) and dissipation and compression work ($N_E \neq 0$, $S = 1$), and the experimental data of reference *3* for air at $N_{Re} = 18{,}578$. Eddy viscosity of Deissler as modified by Hatton et al. (*10*), with $N_{Pr_T} = 1$.

$N_{Re} = 18{,}593$ $\quad N_E = 0.012$ $\quad N_{Pr} = 0.72$

	$S = 0$	$S = 1$
N_{Nu_1}	37.29	37.12
N_{Nu_2}	36.60	36.77
θ_B	1.004	0.995

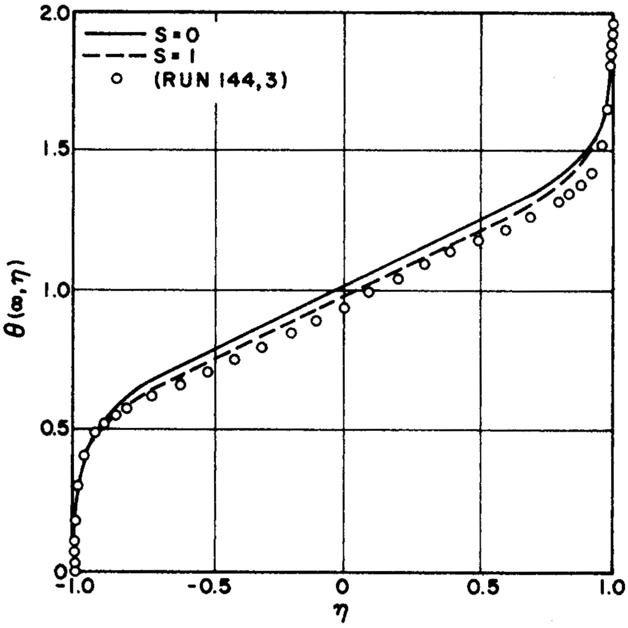

Fig. 5. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures with dissipation ($N_E \neq 0$, $S = 0$) and dissipation and compression work ($N_E \neq 0$, $S = 1$), and the experimental data of reference *3* for air at $N_{Re} = 37{,}528$. Eddy viscosity of Deissler as modified by Hatton et al. (*10*), with $N_{Pr_T} = 1$.

$N_{Re} = 37{,}521$ $\quad N_E = 0.043$ $\quad N_{Pr} = 0.72$

	$S = 0$	$S = 1$
N_{Nu_1}	64.53	63.54
N_{Nu_2}	59.95	61.06
θ_B	1.017	0.98

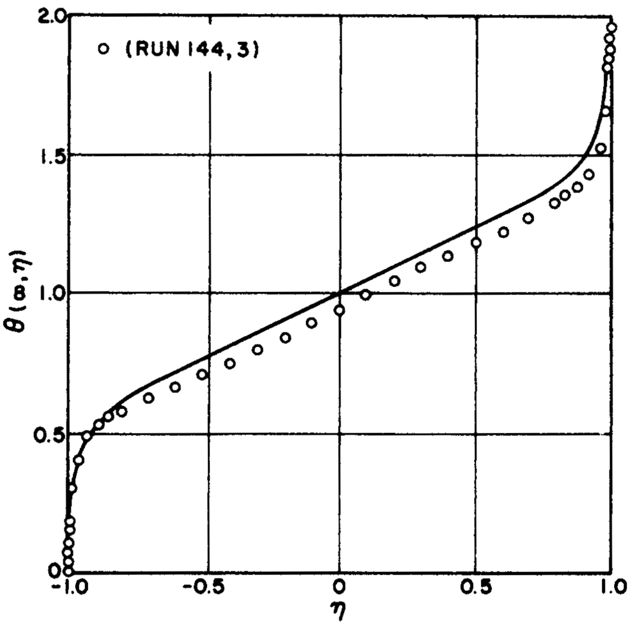

Fig. 6. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures neglecting both dissipation and compression work ($N_E = 0$), and the experimental data of reference *3* for air at $N_{Re} = 37{,}528$. Eddy viscosity of Deissler as modified by Hatton et al. (*10*) with $N_{Pr_T} = 1$.

$N_{Re} = 37{,}571$ $\quad N_E = 0$ $\quad N_{Pr} = 0.72$

$N_{Nu} = 62.28$

$\theta_B = 1.$

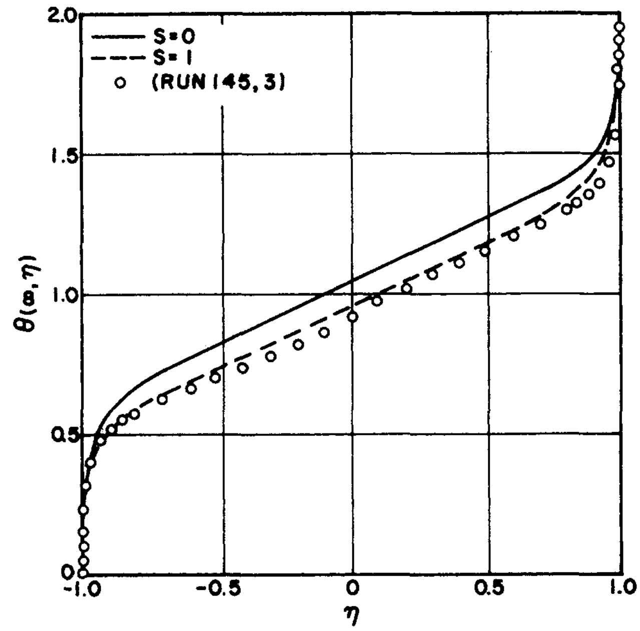

Fig. 7. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures with dissipation ($N_E \neq 0$, $S = 0$) and dissipation and compression work ($N_E \neq 0$, $S = 1$), and the experimental data of reference 3 for air at $N_{Re} = 56{,}689$. Eddy viscosity of Deissler as modified by Hatton et al. (*10*), with $N_{Pr_T} = 1$.

$N_{Re} = 56{,}665$	$N_E = 0.12$		$N_{Pr} = 0.72$
	$S = 0$	$S = 1$	
N_{Nu_1}	92.18	89.44	
N_{Nu_2}	77.78	81.46	
θ_B	1.04	0.953	

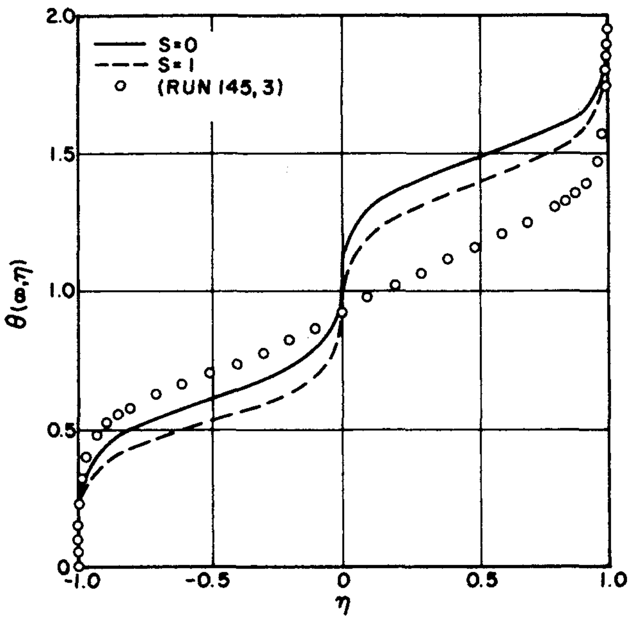

Fig. 8. A comparison between the theoretical fully developed temperature distribution between flat plates with uniform, different wall temperatures with dissipation ($N_E \neq 0$, $S = 0$) and dissipation and compression work ($N_E \neq 0$, $S = 1$), and the experimental data of reference 3 for air at $N_{Re} = 56{,}689$. Eddy viscosity of Deissler not flattened with $N_{Pr_T} = 1$.

$N_{Re} = 56{,}791$	$N_E = 0.12$		$N_{Pr} = 0.72$
	$S = 0$	$S = 1$	
N_{Nu_1}	70.07	65.59	
N_{Nu_2}	54.64	59.96	
θ_B	1.04	0.96	

the velocity field. For air, a velocity of about 330 ft./sec. is taken as the lower limit for the compressibility effects to be negligible. For the experimental set of Corcoran, Sage et al., this would be equivalent to a Reynolds number of about 170,000, which is beyond the range they studied. If the Eckert number is zero, which is equivalent to neglecting dissipation and compression work effects, the temperature profile computed from Equation (3) with an eddy diffusivity which is an even function in $-1 \leq \eta \leq 1$ is a function such that $\theta(\eta) - 1 = 1 - \theta(-\eta)$ for all $-1 \leq \eta \leq 1$, as can be seen in Figure 6. Nonzero Eckert numbers, that is, when dissipation and compression work are accounted for, produce temperature distributions that do not satisfy that condition, as illustrated, for example, by Figure 7. The effects of a nonzero Eckert number for the system discussed here become noticeable only at Reynolds numbers above 30,000.

It is important to note that the analytical temperature distribution deviates from the experimental data to a greater extent when only dissipation is included in the model than when the Eckert number is zero. Note also that the inclusion of only the dissipation term creates a trend in clear conflict with experiment, since experimentally the temperature at the center of the channel consistently appears to be less than the arithmetic mean of the temperatures at the walls.

When both dissipation and compression work are included, Equation (3) leads to temperature distributions that follow the experimental trends, as illustrated for example on the dashed lines in Figures 5 and 7.

The temperature distributions of Figures 3 to 7 were calculated with the eddy diffusivity function of Deissler as modified by Hatton and Quarmby (*10*), who flattened it at its maximum value over the central region of the channel. Figure 8 illustrates the temperature distributions that result from using the unmodified form of Deissler's eddy diffusivity function, which goes to zero at the center of the channel, in the presence of transverse temperature gradients. Clearly the temperature distribution obtained in this case is unacceptable and in marked disagreement with the experimental data, indicating the importance of having a nonzero eddy diffusivity at the center line. These points are discussed in more detail in reference *2*.

CONCLUSIONS

1. The inclusion of dissipation and compression work in the equation of energy for gaseous flows between parallel plates at uniform, different wall temperatures yields temperature distributions which closely agree with the experimental data of Cavers et al. (*3*) if an eddy diffusivity that does not go to zero at the center is employed. In particular, the temperature at the center of the channel is predicted to be less than the average of the wall temperatures, in agreement with the experimental evidence.

2. The net effect of dissipation alone in liquid flows and of dissipation and compression work in gaseous flows is that of increasing the heat flux when the fluid is hotter than the wall, and decreasing it when the fluid is cooler than the wall. This is particularly important when computing eddy conductivities from temperature distributions in channels with uniform, different wall temperatures, because the fluid is heated by one wall and cooled by the other. Thus the heat flux distribution is affected by dissipation and compression work in opposite directions in the two halves of the duct, and the heat flux symmetry about the centerline of the duct that would prevail if dissipation and compression work were negligible is destroyed. This heat flux asymmetry leads to an asymmetric eddy conductivity if the experimental data are interpreted through an integrated form of the energy equation that fails to account for dissipation for

liquids, and both dissipation and compression work for gases.

3. An asymmetrical eddy diffusivity is not compatible with the assumption that the properties of the fluid are independent of temperature, and is a cumbersome concept that makes superposition of solutions impossible. Instead, the inclusion of dissipation and compression work in the energy equation for gaseous flows has been shown to lead to a symmetrical eddy diffusivity. This point is illustrated by comparing Figures 5 and 6, which show temperature distributions obtained with a symmetrical eddy diffusivity function. Note that when dissipation and compression work are included, the theoretical and experimental data are in better agreement than when they are neglected.

4. The contribution of dissipation and compression work to the temperature distribution in the cases studied here is negligible for Reynolds numbers less than 30,000, which correspond to Eckert numbers of less than 0.05.

ACKNOWLEDGMENT

This work was supported by the Office of Naval Research under Contract 5053(01). The work described in this paper was done at both Clarkson College of Technology and Syracuse University.

NOTATION

C_p = heat capacity at constant pressure
C_v = heat capacity at constant volume
f = dimensionless conductivity distribution, $1 + \varepsilon_H/\alpha$
f_o = dimensionless conductivity distribution calculated from experimental temperature data for air, neglecting dissipation and compression work
f_v = dimensionless conductivity distribution calculated from experimental temperature data for air; neglecting compression work only
h = heat transfer coefficient
k = thermal conductivity
N_E = Eckert number, $\dfrac{u_B^2}{C_p \Delta T}$
N_{Nu} = Nusselt number, $\dfrac{4R_e h}{k}$
N_{Pr} = Prandtl number, $\dfrac{\nu}{\alpha}$
N_{Pr_T} = turbulent Prandtl number, $\dfrac{\varepsilon_M}{\varepsilon_H}$
N_{Re} = Reynolds number, $\dfrac{4R_e u_B}{\nu}$
P = pressure
P^+ = dimensionless pressure, $P/\rho u_B^2$
q = heat flux
R_e = hydraulic radius
s = parameter. For gases, $s = 0$ when neglecting compression work and $s = 1$ otherwise. For liquids, $s = 0$ in all cases
T = temperature distribution
ΔT = temperature difference, $T_o - T_{w_1}$
u = velocity distribution
u^+ = dimensionless velocity distribution, $\dfrac{u}{u_*}$
u_* = friction velocity, $\sqrt{\dfrac{\tau_w}{\rho}}$
u_m = velocity at channel center line
u_B = average velocity
$u_B^+ = u_B/u_*$
w = dimensionless velocity distribution, $\dfrac{u}{u_B}$
w^+ = dimensionless velocity distribution $\dfrac{u}{u_m}$
x = axial coordinate
x^+ = dimensionless axial coordinate, $\dfrac{x}{R_e}\dfrac{4}{N_{Re}N_{Pr}} = x\dfrac{k}{R_e^2 u_B \rho C_p}$
y = radial (transverse) coordinate measured from channel centerline
y^+ = dimensionless transverse distance from wall $\dfrac{[R_e - |y|]u_*}{\nu}$
y_m^+ = dimensionless half-thickness, $\dfrac{R_e u_*}{\nu}$

Greek Letters

α = thermometric conductivity, $\dfrac{k}{\rho C_p}$
ε = eddy diffusivity
η = dimensionless transverse coordinate, $\dfrac{y}{R_e}$
θ = dimensionless temperature distribution, $\dfrac{T - T_{w1}}{\Delta T}$
θ_w = dimensionless wall temperature difference, $\dfrac{T_{w2} - T_{w1}}{\Delta T}$
μ = viscosity
ν = kinematic viscosity, μ/ρ
ρ = density
τ_w = shear stress at wall
$\Lambda_1 = \left[\int_{-1}^{1} \frac{1}{f} d\eta\right]^{-1}$

Subscripts

B = cup mixing average
COMP = due to compression work
DISS = due to dissipation
EXP = measured experimentally or derived from experimental data
m = maximum or center line value
o = entrance of channel, $x = 0$
w_1 = at lower wall of channel
w_2 = at upper wall of channel

LITERATURE CITED

1. Bird, R. B., W. E. Stewart, and E. N. Lightfoot, "Transport Phenomena," Wiley, New York (1960).
2. Blanco, J. A., Ph.D. dissertation, Syracuse Univ., Syracuse (1967).
3. Cavers, S. D., N. T. Hsu, W. G. Schlinger, and B. H. Sage, *Ind. Eng. Chem.*, **45**, 2139 (1953).
4. Corcoran, W. H., B. Roudebush, and B. H. Sage, *Chem. Eng. Progr.*, **43**, 135 (1947).
5. Corcoran, W. H., F. Page, W. G. Schlinger, and B. H. Sage, *Ind. Eng. Chem.*, **44**, 410 (1952).
6. Corcoran, W. H., and B. H. Sage, *AIChE J.*, **2**, 251 (1956).
7. Esghy, S., *Intern. J. Heat Mass Transfer*, **7**, 1340 (1964).
8. Gill, W. N., and M. Scher, *AIChE J.*, **7**, 61 (1961).
9. Harrison, W. B., and J. R. Menke, *Trans. ASME*, **71**, 797 (1949).
10. Hatton, A. P., and A. Quarmby, *Intern. J. Heat Mass Transfer*, **6**, 903 (1963).
11. Hsu, N. T., K. Sato, and B. H. Sage, *Ind. Eng. Chem.*, **48**, 2218 (1956).
12. Madejsky, J., *Intern. J. Heat Mass Transfer*, **6**, 49 (1963).
13. Page, F., W. G. Schlinger, D. K. Breaux, and B. H. Sage, *Ind. Eng. Chem.*, **44**, 419 (1952).
14. Sage, B. H., *AIChE J.*, **5**, 331 (1959).
15. Schlichting, H., "Boundary Layer Theory," McGraw-Hill, New York (1960).
16. Schlinger, W. G., N. T. Hsu, S. D. Cavers, and B. H. Sage, *Ind. Eng. Chem.*, **45**, 864 (1953).
17. Tyagi, V. P., *J. Heat Transfer, Trans. ASME Ser. C*, **88**, 161 (1966).
18. *Ibid.*, **89**, 1321 (1966).
19. Venezian, E., and B. H. Sage, *AIChE J.*, **7**, 688 (1961).

Manuscript received June 30, 1969; revision received October 31, 1969; paper accepted November 6, 1969.